Adaptive Fuzzy Logic Controllers with Sensitivity Learning Rules

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Abstract

This paper presents a design sensitivity method for learning rules of adaptive Fuzzy Logic Controllers (FLCs). FLCs can be made to adapt their initial knowledge bases to the changes in the operating conditions. With a model reference control strategy, the performance index is defined as the error function between the desired and actual outputs of the process. It is to be minimized by the gradient-based optimization which acquires a knowledge of the sensitivity of the performance index with respect to the parameters of FLCs. The tracking problem of a continuous stirred tank reactor is investigated as a realistic non-linear case study for the viability of the proposed methodology.

1. Introduction

Up to now, FLCs have been widely used in many applications of process control. Compared to conventional control strategies, FLCs can yield superior controller performance for complex and non-linear processes. This is because their linguistic rules are backed up by the experience and intuition of the operators. This acquired knowledge is initially provided by interviews with the operators in natural language. For example, if a small positive error between the desired and actual output occurs then the FLC output should be small positive. However, the fuzzy rules require more precise rules to yield a better control performance. The fine-tuning of FLC is a trial-and-error process that may involve quantitatively adjusting many parameters in FLC laws.

In adaptive control methods, the parameters of controllers are systematically altered on-line in response to a measured performance value. Two kinds of typical controllers are usually considered: a Self-Tuning Controller (STC) and a Model Reference Controller (MRC). The STC is generally based on two steps. First, the model of the process is estimated in real time and then controller parameters are adjusted to improve the control performance according to the recursive model-parameter estimates[1]. For the MRC[2], the performance index of the control system is generally defined as the difference between the model reference output and the process output. The error minimization is to be carried out to tune the controller parameters.

In this work, MRC class is adapted as an inverse modeling technique of a non-linear process for a learning algorithm of the FLC in order to refine the rule based knowledge when changes in the process may occur. The performance index is defined in terms of the difference between the desired and the actual output of the process. The sensitivity method[3] is implemented to compute the gradient of the performance index with respect to the FLCs' parameters. With gradient descent-based optimization, the parameters are adjusted so that the FLC output allows a better tracking of the desired output. In other words, the FLC can be made to adapt its initial knowledge base to obtain fine tuning when changes in operating conditions take place. The performance of the proposed technique is presented here by regulating a product concentration in a continuous stirred tank reactor.

2. Fuzzy logic control laws

Mathematical expression for a class of fuzzy logic control laws of Langari[4] is reviewed first. A general rule of proportional plus derivative control law can be expressed as follows:

If the error e is X_i ***and*** the rate of change in error is Y_j ***then*** the controller output u is $Z_{i,j}$.
where X_i is the i^{th} set for the fuzzy variable e, Y_j is the j^{th} set for the fuzzy variable $\dot{e}$ and Z is $(i,j)^{th}$ set for the fuzzy variable u.

According to the definition of membership functions in Fig. 1, the controller output is determined by the compositional rule of inference[5] as:

$$u=\frac{\mu_{i,j}U_{i,j}+\mu_{i+1,j}U_{i+1,j}+\mu_{i,j+1}U_{i,j+1}+\mu_{i+1,j+1}U_{i+1,j+1}}{\mu_{i,j}+\mu_{i+1,j}+\mu_{i,j+1}+\mu_{i+1,j+1}} \tag{1}$$

where the membership values of the rules, following the product t-norm, are obtained by

$$\begin{aligned}\mu_{i,j}&=\mu_i\mu_j\\ \mu_{i+1,j}&=\mu_{i+1}\mu_j\\ \mu_{i,j+1}&=\mu_i\mu_{j+1}\\ \mu_{i+1,j+1}&=\mu_{i+1}\mu_{j+1}\end{aligned} \tag{2}$$

From Eqs. (1)-(2), the fuzzy logic control laws u can be written as:

$$\begin{aligned}u(e,\dot{e},\vec{w})=&[1-s_i(e-E_i)][1-s_j(\dot{e}-\Delta E_j)]U_{i,j}\\ &+[1-s_i(E_{i+1}-e)][1-s_j(\dot{e}-\Delta E_j)]U_{i+1,j}\\ &+[1-s_i(e-E_i)][1-s_j(\Delta E_{j+1}-\dot{e})]U_{i,j+1}\\ &+[1-s_i(E_{i+1}-e)][1-s_j(\Delta E_{j+1}-\dot{e})]U_{i+1,j+1}\end{aligned} \tag{3}$$

where $\vec{w}$ is a vector of which elements are FLC parameters, i.e., s, E, ΔE and U.

In Eq. (3), the strongly non-linear proportional plus derivative PD-type fuzzy logic laws have been presented. In general, the integral action $\int edt$ (I) can be substituted or added into Eq. (3) in order to yield PI-type or PID-type fuzzy logic control laws respectively.

3. Sensitivity Method for Learning Rules

Fig. 2 shows the typical diagram of controlling the process and learning the inverse system of the process simultaneously. When the learning module is considered, the FLC model in Eq. (3) can be expressed by the function in the terms of FLC parameters.

$$u(t)=u(\vec{w}(t)) \tag{4}$$

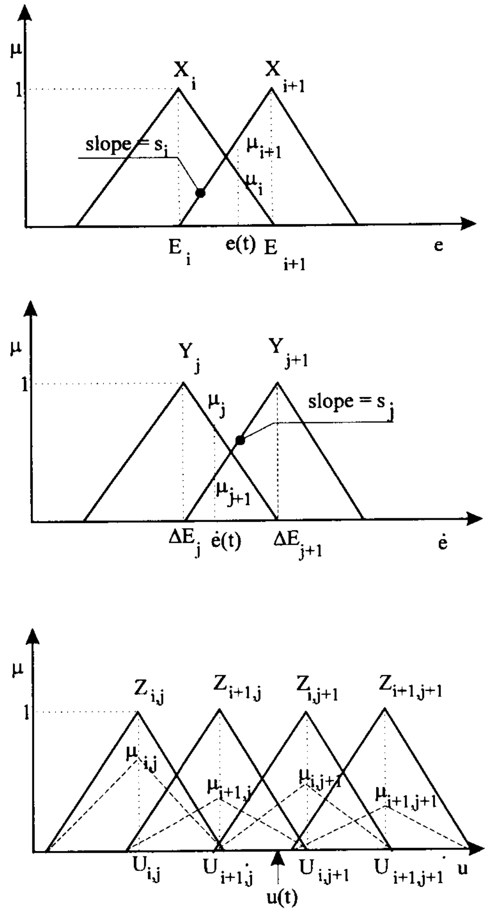

Fig. 1 Membership functions for PD-type FLCs.

In controlling module, FLC will normally use error e, which is the difference between the output of the process y and the reference input y_d, to control the process according to their rule-based knowledge when the steady state process input u_s can not maintain the process output at the reference point. In learning module, the FLC parameters in Eq. (4) is to be adjusted such that FLC output u is capable of controlling the process so as to minimize the error ε between the process output y and the reference model output $y_{\tilde{d}}$. This part is to gradually refine the initial rule-based knowledge as the FLC samples new cases.

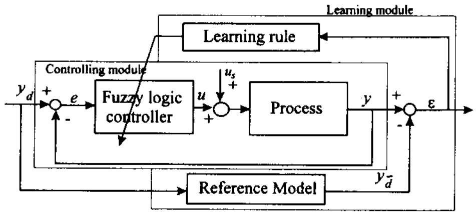

Fig. 2 Diagram of Adaptive FLC system.

For convenience in this discussion, the single-input single-output plants are assumed to be static as $y = f(u)$. The dynamic case will be discussed in the next section. Now, the performance index $J(\vec{w})$ is defined as the function of the difference between an output of a process y and a reference model output $y_{\tilde{d}}$ as:

$$J(\vec{w}) = f(\varepsilon(y)) \qquad (5)$$

A gradient descent-based optimization is implemented to adapt the FLC parameters. The gradient of the performance index with respect to the FLC parameters $\Delta\vec{w}$ is defined as Eq. (6).

$$\Delta\vec{w} = \frac{\partial J}{\partial \vec{w}} \qquad (6)$$

In Eq.(7), the negative gradient of the performance index indicates the descent direction to update the FLC parameters while the magnitude of learning rate is specified by the step-size δ.

$$\vec{w}(t+1) = \vec{w}(t) - \delta(t)\Delta\vec{w}(t) \qquad (7)$$

where t is the time index.

Hence, the FLC parameters can be adjusted by sensitivity learning rules in Eq. (7) so that the FLC must produce an output which decreases the value of the performance index. Now, a design sensitivity analysis will be implemented in order to determine the gradient of the performance index.

Let's apply the derivative chain rule in order to obtain the gradient quantity as:

$$\Delta\vec{w} = \frac{\partial J}{\partial \vec{w}} = \frac{\partial J}{\partial y}\frac{\partial y}{\partial u}\frac{\partial u}{\partial \vec{w}} \qquad (8)$$

After applying direct differentiation method [3], the quantity of $\partial J/\partial y$ is directly given from Eq. (5). The sensitivity function $\partial u/\partial \vec{w}$ of FLC can be calculated from the model of Eq. (4). Now, consider the sensitivity function of $\partial y/\partial u$. It is to be computed from the static or dynamic function of the process or, at least, the sign of its sensitivity function $sign(\partial y/\partial u)$ is required to be known if we have some qualitative knowledge of the process.

4. Simulations

The control of the product concentration in continuous stirred reactor[6] is applied to investigate the viability of on-line sensitivity learning rules. The dynamics of the process can be represented by a set of non-linear differential equations

$$\dot{c}(t) = \frac{q}{v}(c_0 - c(t)) - k_0 c(t) e^{-\frac{E}{RT(t)}} \qquad (9)$$

$$\dot{T}(t) = \frac{q}{v}(T_0 - T(t)) + k_1 c(t) e^{-\frac{E}{RT(t)}} + k_2 q_c(t)(1 - e^{-\frac{k_3}{q_c(t)}})(T_{c0} - T(t)) \qquad (10)$$

Within the reactor, two chemicals are mixed and they react to produce a product compound at the concentration c and the temperature T. This exothermic reaction can be slowed down by introducing the coolant at flow-rate q_c. By regulating q_c, the temperature can be varied and then the product concentration can be controlled. c_0 is the inlet feed concentration, q is the flow rate of the process, T_0 and T_{c0} are the inlet feed and coolant temperature respectively, v is the of volume of reactor, k_0, k_1, k_2 k_3 and E/R are the thermodynamic constants. Their numerical values are given in the Appendix.

Fig. 3 shows the open-loop response due to the exciting input in the form of step signals. It can be noted that the overshoot magnitude and the damping of the process output vary considerably, depending on the operating conditions.

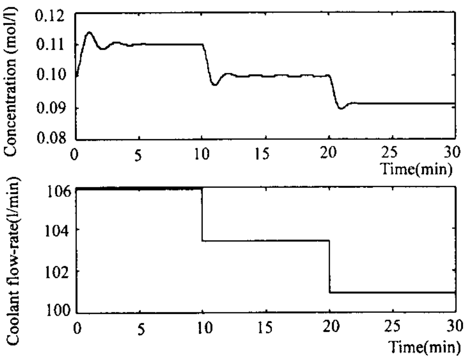

Fig. 3 Open-loop step response.

In this case study, the P-type control laws of FLC are developed from Eq.(3) as follows:

$$u = [1 - s_i(e - E_i)]U_i + [1 - s_i(E_{i+1} - e)]U_{i+1} \quad (11)$$

for i=1,2

The rule-based knowledge is defined as follows.

If the error is negative E_1, ***then*** the controller output is negative U_1.
If the error is zero E_2, ***then*** the controller output is center U_2.
If the error is positive E_3, ***then*** the controller output is positive U_3.

The membership functions of FLC are given in Fig 5. These proportional fuzzy logic control laws are applicable to the operating condition c=0.1 mol/l and q_s=103.41 l/min where

$$q_c = q_s + u \quad (12)$$

To illustrate the effectiveness of the proposed methodology, the operating conditions are changed to c=0.09 mol/l while the desired output of the reference model is a step response of the first-order element $c_{\tilde{d}}$ in Fig 6.

Without the sensitivity learning rules, FLC can not alter the product concentration of reaction to the desired concentration as can be seen from Fig. 6. The steady state error is approximately -0.005 mol/l

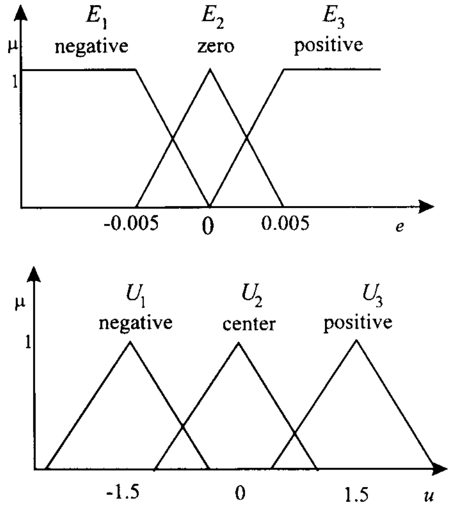

Fig. 5 Membership functions of P-type FLC.

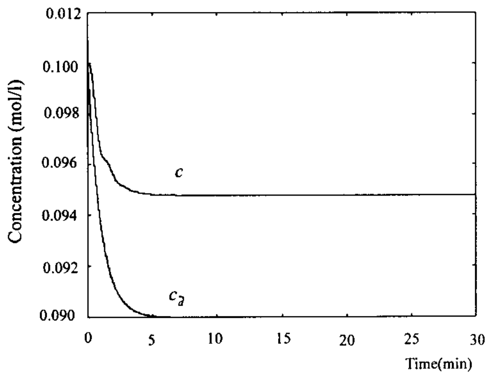

Fig. 6 Step response without sensitivity learning rules.

To make refinements of the initial rule-based knowledge, define the performance index as:

$$J(\vec{w}(t)) = \frac{1}{2}(c(t) - c_{\tilde{d}}(t))^2 \quad (13)$$

where $\vec{w} = [U_1, U_2, U_3]^T$

In this case, the sensitivity learning rules in Eq. (7) allow adjustment to the centers of the sets for the fuzzy variable u. Note that differentiation of the Eqs (9) and (10) yields the sensitivity model of the process which is required for the sensitivity learning rules in Eq. (7) and can be written in Eq (14) and (15) respectively.

$$s_c \Delta \frac{\partial c}{\partial q_c} \qquad s_T \Delta \frac{\partial T}{\partial q_c}$$

$$\dot{s}_c = -k_0 c_a \frac{E}{RT^2} e^{-\frac{E}{RT}} s_T - s_c\left(\frac{q}{v} + k_0 e^{-\frac{E}{RT}}\right) \qquad (14)$$

$$\dot{s}_T = -\frac{q}{v} s_T + k_1 s_c e^{-\frac{E}{RT}} + k_1 \frac{E c_a s_T}{RT^2} e^{-\frac{E}{RT}} - k_2(1 - e^{-\frac{k_3}{q_c}}) q_c s_T + k_2 (T_{c0} - T) - k_2 (1 + \frac{k_3}{q_c}) e^{-\frac{k_3}{q_c}} (T_{c0} - T) \qquad (15)$$

Fig 7. shows FLC can adapt itself to follow the reference output to reach the desired concentration with the sensitivity learning rules.

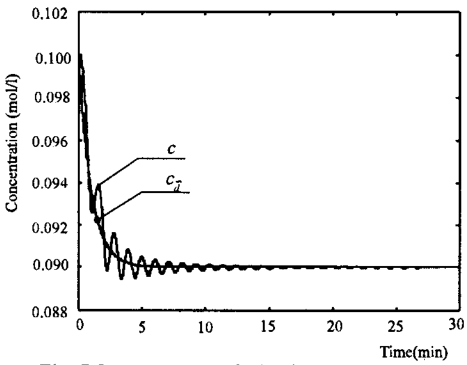

Fig. 7 Step response of adaptive FLC system.

In Fig 8, the adaptive FLC shows its capability to track the sinusoidal output of the reference model after 5 min.

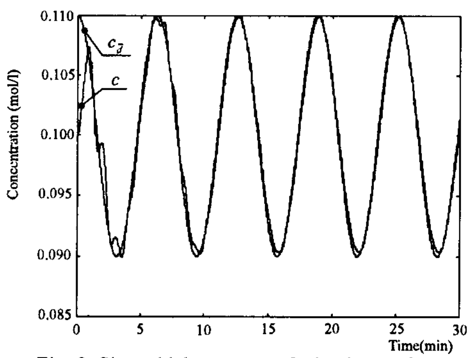

Fig. 8 Sinusoidal response of adaptive FLC system.

5. Conclusion

An adaptive FLC with sensitivity learning rules has been presented in this paper. Simulation results showed that the proposed methodology was successfully applied to the control of the product concentration in a continuous stirred reactor

6. Appendix

Parameter	Numerical value
q	100 l/min
v	100 l
k_0	$7.2\text{x}10^{10}$ min^{-1}
E/R	$1\text{x}10^4$ K
T_0	350 K
T_{c0}	350 K
ΔH	$-2\text{x}10^5$ cal/mol
C_p, C_{pc}	1 cal/g/K
ρ_p, ρ_{pc}	1000 g/l
h_a	$7\text{x}10^5$ cal/min/K
c_0	1 mol/l

$$k_1 = -\frac{\Delta H k_0}{\rho C_p} \quad k_2 = \frac{\rho C_{pc}}{\rho C_p v} \quad k_3 = \frac{h_0}{\rho_c C_{pc}}$$

7. References

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