The Implementation of Polya's Model in Solving Problem-Questions in Mathematics by Grade 7 Students

Satyaprakash Gopinath[1*] and Supinda Lertlit[2]

[1*]*Rangsit University, Thailand*
[2]*Faculty of Education, Rangsit University, Thailand*

ABSTRACT

This research reports a study of the difficulties the students face in problem solving questions in mathematics and how to tackle it by using George Polya's four-step problem solving model. The objectives of this research are 1) to develop the solving problem-questions skills in mathematics by using George Polya's model for grade 7 students, and 2) to evaluate students' achievements in mathematics on problem solving questions after using Polya's problem solving model.

A mixed of quantitative and qualitative methodologies were conducted in this study. The subjects of this study were a group of ten grade 7 students of an international school. The research tools were a pre-test before the treatment and a post-test after the treatment. The test included ten questions of problem solving which contained five "multiple choice questions" and five "long answer questions" for reading comprehension purpose. The data was collected and compared by quantitative analysis using mean, percentage and standard deviation, and by qualitative content analysis.

The results from the pre and post tests were obtained, analyzed, and compared to find the improvement of the students' performances while answering the problem solving questions. This study concludes with suggestions of methods that teachers can use to help the students overcome their difficulties in problem solving questions in mathematics.

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* *Corresponding author*
E-mail address: satyaprakash.g57@rsu.ac.th

บทคัดย่อ

งานวิจัยฉบับนี้แสดงถึงการศึกษาปัญหาของนักเรียนในการแก้โจทย์วิชาคณิตศาสตร์และการแก้โจทย์โดยใช้รูปแบบการแก้ปัญหาแบบ 4 ขั้นตอนของ โพลยา จุดประสงค์ของงานวิจัยฉบับนี้คือเพื่อพัฒนาทักษะการแก้โจทย์วิชาคณิตศาสตร์สำหรับนักเรียนชั้นมัธยมศึกษาปีที่ 1 ด้วยวิธีของโพลยา และ เพื่อประเมินประสิทธิภาพของนักเรียนในการแก้โจทย์หลังจากใช้รูปแบบการแก้ปัญหาของโพลยา

ในการวิจัยครั้งนี้ ผู้วิจัยใช้เครื่องมือวิจัยเชิงปริมาณและเชิงคุณภาพ โดยกลุ่มตัวอย่างคือนักเรียนชั้นมัธยมศึกษาปีที่ 1 จำนวน 10 คน เครื่องมือวิจัยที่ใช้คือแบบทดสอบก่อนเรียนและแบบทดสอบหลังเรียนแบบทดสอบประกอบด้วยโจทย์ทั้งหมด 10 ข้อ แบ่งเป็นแบบทดสอบอัตนัยสำหรับการอ่านเพื่อความเข้าใจ 5 ข้อ และแบบทดสอบปรนัย 5 ข้อ ผลการทดสอบที่ได้จะถูกนำมาเปรียบเทียบโดยใช้ค่าเฉลี่ย อัตราร้อยละ ส่วนเบี่ยงเบนมาตรฐาน และการวิเคราะห์เนื้อหาเชิงคุณภาพ

ผู้วิจัยนำผลที่ได้จากการทดสอบก่อนเรียนและหลังเรียนมาวิเคราะห์และเปรียบเทียบเพื่อหาพัฒนาการของนักเรียนในการแก้โจทย์ การศึกษาครั้งนี้สรุปได้ว่าผู้สอนนั้นสามารถใช้วิธีการที่หลากหลายในการช่วยให้นักเรียนมีพัฒนาการในการแก้โจทย์วิชาคณิตศาสตร์

Introduction

Mathematics remains an enigma for many students and making the subject endearing to the students is always a challenging task for the teacher or parents. Experience and studies show that solving problem questions in mathematics are difficult for many young students. As a mathematics teacher, the researcher agrees that the educators face enormous number of challenges in today's teaching towards students. Many researchers have found that the main area that must be looked over where most of the students find it difficult in mathematics is "problem solving questions" (Carpenter, 1989).

Word problems can be of various types. According to Anne Roche a problem can have "any mathematical problem written in a sentence or sentences." A word problem does not only include a single mathematical calculation, but multiple calculations as well. The information given might be less or more than expected by the problem solver. Roche (2013) has importantly highlighted the problems being faced by students during word problem solving.

It should be remembered that all other mathematics problems, word problems also need several strategies bound together for solving the problem. Some students enjoy mathematics more than any other subjects because it has less literature involved. These students, when faced with word problem questions are really all at sea in case of finding the answers. The word problems are like mountains to them and hence are very frustrated. It might sometimes be very difficult for a teacher to judge whether the student is unable to comprehend the problem or it is just a calculation mistake. Computation as well as comprehension skills are both needed for solving word problem questions.

Selected Literature Review as Background of the Study

1. Problem Solving Heuristics

Schoenfeld, A. H. (1985) states that theories of mathematics problem solving have placed a major focus on the role of heuristics. Surely it seems that providing explicit instruction on the development and use of heuristics should enhance problem solving performance; yet it is not that simple. According to Polya (1957), problem solving is a type of skill that is practical. He says that students will learn problems when they observe and imitate what other people do while solving any similar problem. Polya distinguished problem solving into 4 different models/phases. The first is to understand the problem so the task can be seen clearly, the second is to devise a plan, the third is to carry out the plan and the fourth is to look back and check the completed equation or solution.

2. Mathematical Induction

Induction is the combination of particular instances and the process of discovering general laws by observation (Polya, 1962). Induction is not only used in all sciences, but also vastly used in mathematics. Mathematical induction is mainly used to prove theorems of a certain kind such as "the sum of consecutive cubes is equal to the square of the sum". An example of this mathematical induction is as follows:

$$1 + 8 + 27 + 64 = 100$$
$$(1)^3 + (2)^3 + (3)^3 + (4)^3 = (10)^2$$

3. Different Types of Problems

Well-structured problems: Well-structured problems are types of questions that have a known solution for the variable or equation and usually provide all the necessary information for the solver to come to an answer (Jonassen, 1997). In other words, well-structured problems yield the right answer by the usage of an appropriate algorithm. These kind of structured problems are usually arranged in a predictable manner and require very limited usage of concepts.

Ill-structured problems: Ill-structured problems are the types of questions that may have many solutions or in some instances, no solution at all. Ill-structured problems have the chance of integrating multiple concepts and might provide very little information about the concepts relating to the problem. Since ill-structured problems have unclear goals and incomplete information (Voss, 1988), they have multiple solution paths so that the students are required to make multiple decisions about the problem but have many questions in their minds such as; What is the goal? How can I get to the final solution?

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What concepts are acquired to solve the problem? Does the final solution meet the goal of the problem?

Therefore if solving problem questions can be successfully taught in the classroom, the students will also be successful in applying them in real life situations. This will also enable the students to internalize the theories learnt from school becoming meaningful to them.

4. Different Procedures in Problem Solving

There are many ways to do the problem questions solving, the followings are some examples of researchers who created different procedures to solve problem questions.

Polya (1945) created a four steps principle for problem solving. The four steps that George Polya used for solving problem-questions are as follows.

First principle: *Understand the problem*

Second principle: *Devise a plan*

Third principle: *Carry out the plan*

Fourth principle: *Look back*

According to Bhatia (1992), using the four-step problem is a very clever and clear approach for every student to solve any typical problem. These steps include

1) Basic concepts of the problem and understanding it
2) Crafting a plan to sort it out
3) Following every step of the plan without fail
4) Cross-checking it. The last step is very vital.

Aim of the Study

The objectives of this study are:

1) To compare students' achievement on problem-solving in mathematics of grade 7 students before and after using Polya's problem solving model.

2) To explore the way the student's apply Polya's problem solving model.

Terminology

Solving problem-questions refers to the process of finding solutions to difficult or complex issues in any mathematics exercise where the situation is explained in words.

Polya's Model refers to one of the problem solving techniques which consists of four phases of 1) Understanding the problem, 2) Devising a plan, 3) Carrying out the plan, and 4) Looking back.

Research Methodology

The study made use of both qualitative and quantitative research methods to find the difficulties students face while solving problem-questions and how Polya's problem solving model will have effects on their solutions.

1. Participants

The population for this study was 150 students of Global Indian International School. The purposive sampling method was used to select ten students of grade 7 as the subjects of this study.

2. Research Instruments

The instruments of this study were pretest and posttest consisted of ten items including five multiple choice questions and five long answer questions. The questions were consisted of both well-structured and ill-structured items. The multiple choice questions were given two marks each where the long answer questions were given three marks each. The researcher also took into account the usage of Polya's model in each answer of the students. The researcher used 20 periods for this study where each period was taught using Polya's word problem solving model.

3. Data Collection Procedures

In data collection process, the researcher adopted a weekly plan method which is a 50 minute-period of the researcher's mathematics class. The researcher adopted testing methods in order to evaluate the progress of each student. The test paper contained both open-ended and close-ended types of questions where each student was able to exhibit his/her way of answering the questions and choosing the correct answer from the given options. After the students' test papers were corrected, the results of the tests were compared for analysis.

A pretest was given to the students in the month of August and post-test was given to the students in the month of September 2015. The tests were conducted before and after using Polya's word problem solving model respectively. The marks obtained by the students at the tests was compared and ascertained using the t-test by using standard deviation and mean average to see the quantum of improvement in the students after they have been exposed to Polya's word problem solving model.

Data Analysis

In this research, a descriptive statistical analysis was used to calculate the basic statistic values such as percentage, mean, and standard deviation of the collected data in order to explore different types of data. The researcher also used a comparative analysis of the outcome of students at different levels. The following evaluation criteria were used to compare the results obtained before and after the tests.

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Score range	Description
21 - 25	Outstanding
16 - 20	Very Satisfactory
11 - 15	Satisfactory
6 - 10	Poor
0 - 5	Very poor

The students' performances were compared by using mean average and standard deviation in order to check where each student stands while comparing to the class as a whole and the percentage was used to classify on which grade level each student stands and the percentage was used to classify on which grade level each student stands.

Results and Discussion

This research has resulted in two major findings.

1. Student's Pre-test & Post-test Performance in Solving Word Problem questions

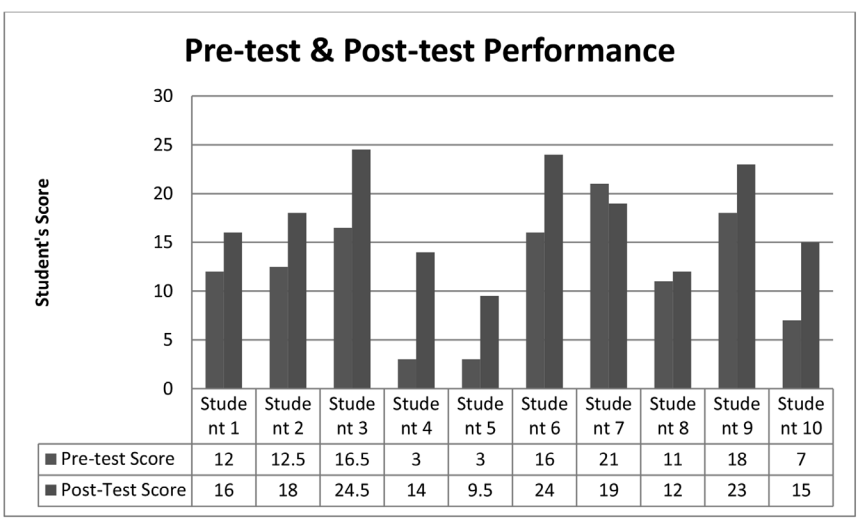

Figure 1. Students' Pre-test and Post-Test Scores.

Figure 1 shows the student's scores on the pre-test and post-test they took. The table above shows that only one student got an outstanding result in the pre-test compared to three students in the post-test. Two students were found below the very poor category in the pre-test where none of the students were in the very-poor category in the post test and four students were in the above satisfactory

category in the pre-test whereas six students were in the above satisfactory in the post-test. There was only one student (Student 7) whose scores decreased and the reason was due to the amount of careless mistakes the student made.

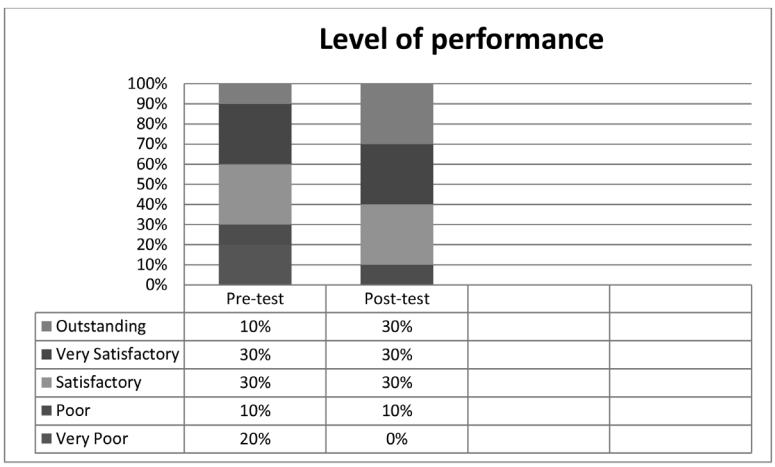

Figure 2. Level of Performance on the Pre-Test.

In Figure 2, the level of performance shows that 30% of the students are below satisfactory on the pre-test compared to only 10% on the post-test. 40% of the students were found to be above satisfactory level in the pre-test where 60% were found to be above satisfactory level in the post-test. In overall, the improvement of the students' performance in the word problem solving level arose significantly after the input of Polya's Model in the treatment after the pretest.

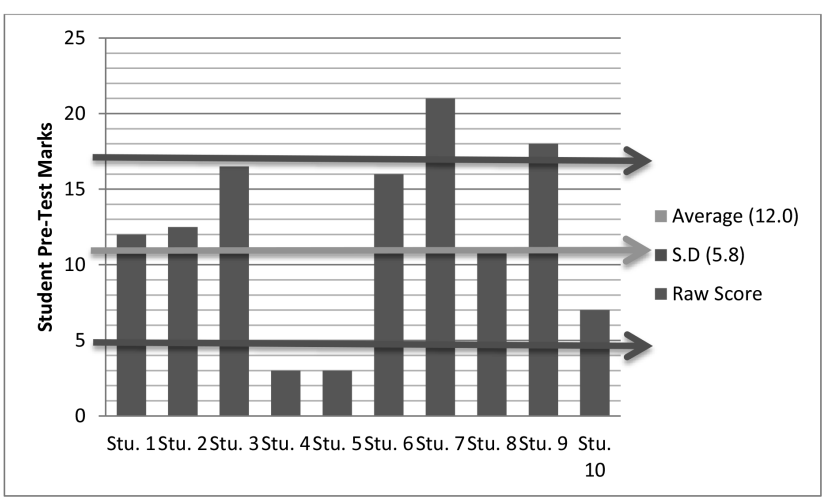

Figure 3. Pre-Test Mean/S.D

From the table above it is seen that grade 7 are not performing well, though some of their scores are well above the mean average. Students 4 and 5 are the under performers here. The chart highlights average understanding of the subject by students across a wide spectrum of classes. This is the overall picture in the pre-test with the scores suggestive of the fact that the students were not clear in their understanding of worded mathematics problems.

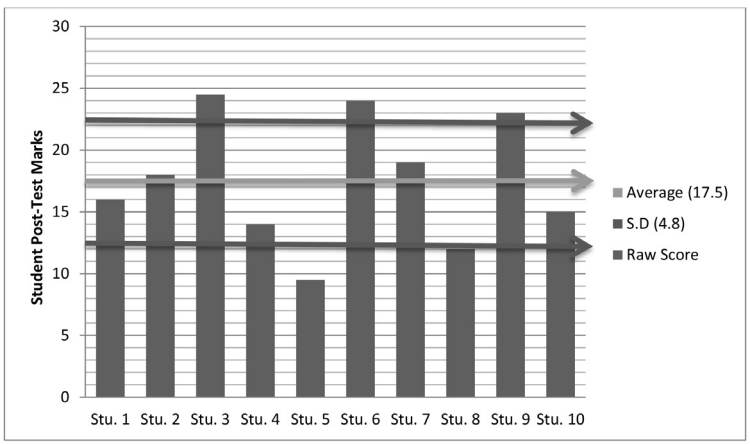

Figure 4. Post-Test Mean/S.D

The scores obtained by the students in the post –test reflects a different picture of student perceptions and understanding of solving problem questions. The average of the class has gone up substantially and the variations from the standard -deviation are well within control. There are a number of good performers and only one student is under performing (due to simple mistakes during the test). The conclusion that can be drawn is that post test scores reflect better understanding of solving problem questions.

2. Common Mistakes Made by Students

The scanned answer sheets from six students' works showing difficulties in a common question are shown in Figures 5 – 10 as follows:

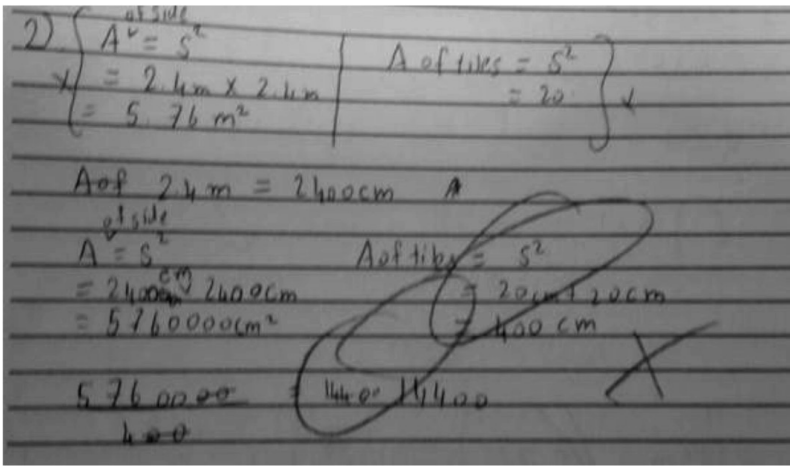

Figure 5. The #1 student's answer sheet

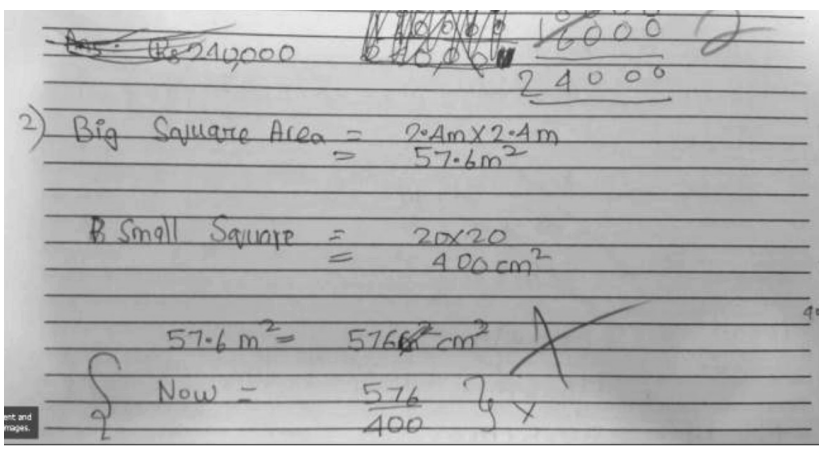

Figure 6. The #2 student's answer sheet

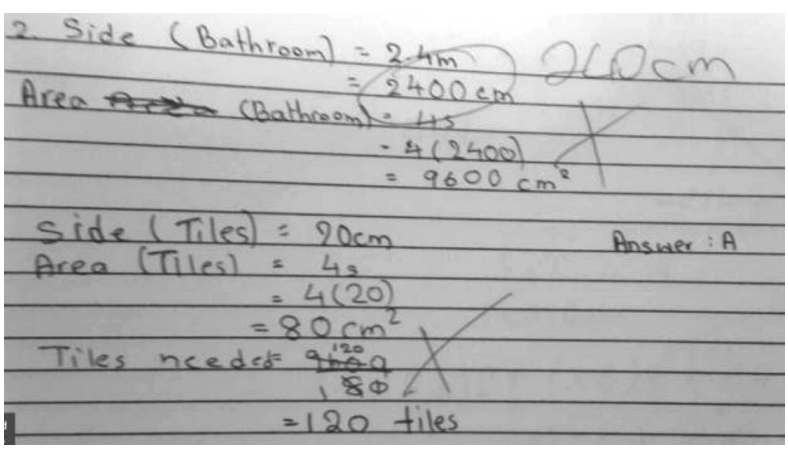

Figure 7. The #3 student's answer sheet

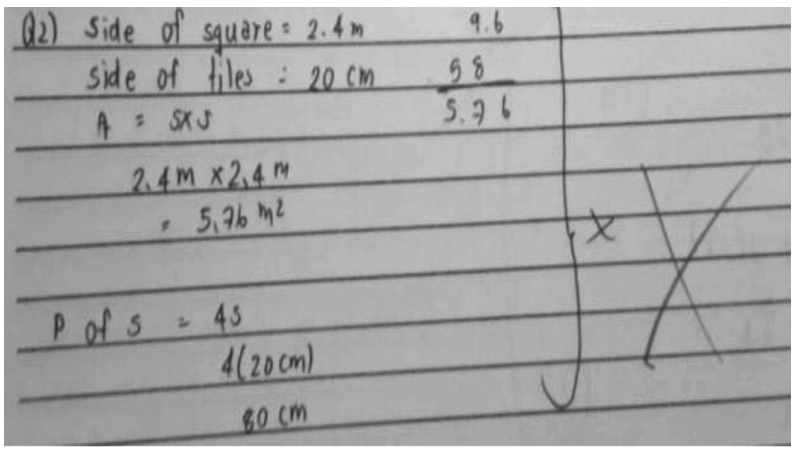

Figure 8. The #4 student's answer sheet

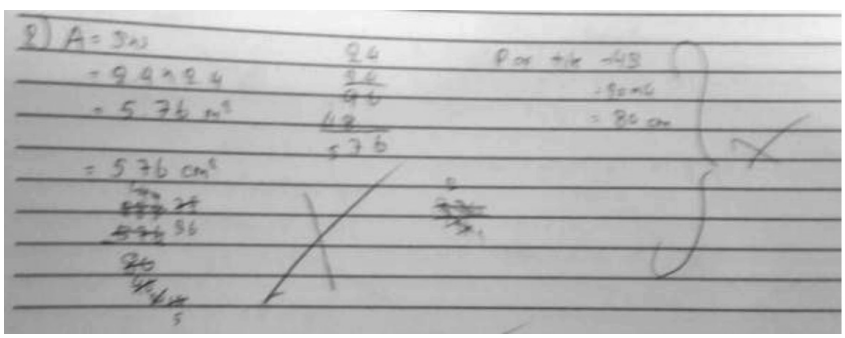

Figure 9. The #5 student's answer sheet

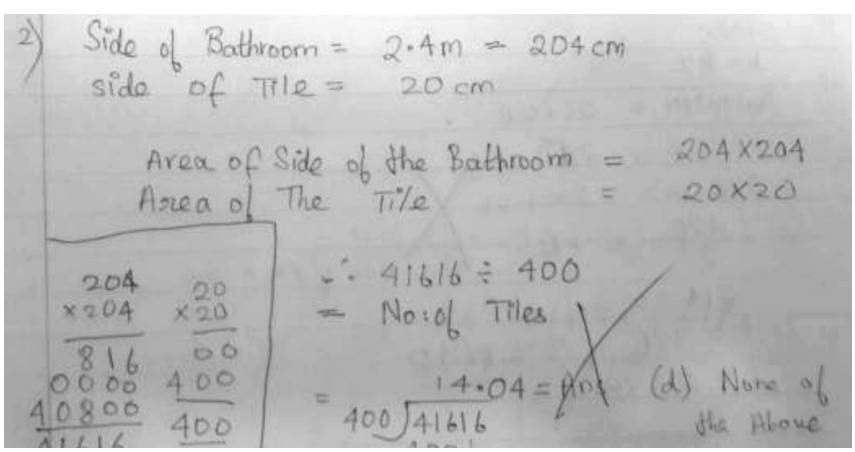

Figure 10. The #6 student's answer sheet

From Figures 5 – 10, six out of ten students who made incorrect answers in question no.2 of the multiple choice question made common mistakes on not converting the units before solving for the question. What the students should have done is to convert the side of the square bathroom into "cm" before finding the area of it then further on dividing the area of the bathroom with the area of the tiles to get the number of tiles in total. Out of these six students, five of them made mistakes on not converting the units and one found the perimeter instead of finding the area.

According to Fajemidagba and Olawoye (2009), their research findings suggested that the student' belief about mathematics and mathematical problem solving was significantly related to their performance in mathematics. It was therefore recommended that mathematics teachers should employ new instructional strategies that are capable of improving students' beliefs about mathematics and mathematical problem solving. Incorporating Polya's problem solving method, Yuan (2013) confirmed that the four steps of Polya's method help students keep in mind the common sense nature of math and mathematical problem solving. With them, students were able to use the reasoning abilities they already have to leap hurdles they might have previously thought insurmountable. The mathematics teachers should implement this method, properly introduce and explain to their students how it can be used in basic skills mathematics classes to ease the students' fears of math, and potentially change their common misconceptions of the subject to leveraging the performance ultimately in the future.

Conclusions

The findings based on George Polya's model conducted on students of grade 7 bring out some conclusions. The findings highlight the fact that students performance based on solving problem questions in mathematics drastically improved using George Polya's four-step model and distinct patterns were observer in marks obtained by the students in the Pre-test and Post-test.

The finding of this study also suggests a teaching on mathematical problem solving models with a variety of math problem steps which requires the students to think analytically by trying the model to solve the question, having Polya's problem-solving model makes solving problem questions easy to learn and become efficient problem solvers. As stated by Hugar (2011), the researcher also believes that in order for a student to be good at solving problem questions he/she should practice and master it periodically. The teachers' also should try to adapt new methods than using the traditional method in every days class Burns (2010).

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Solving problem questions in mathematics often has been seen as contradicting in achievement levels of students to normal problems just using mathematical formulae. Solving problem questions require a better understand on the inter-relations among various aspects of the problem. This required a good understanding of the contents of the chapter as well as a clear understanding of the language which connects various parts of the problem.

The reason why there were few questions that many of the students made mistakes irrespective of the usage of Polya's model is because of the background knowledge in the topic that came in the test. It is crucial that content instruction along with Polya's model go parallel with each other. In other words, the students need to understand the background of the topic in order to manipulate the given information.

The comparison of the Pre-test and Post-test scores bring out the sharp increase in the number of students whose results were outstanding. Similarly the Post-test results indicate no student falling under Very Poor category and indicate 20% increase in the outstanding category. The other categories maintain the same levels, which is somewhat surprising.

Some simple calculation errors made by students, such as failing to convert units resulted in lessening of scores of some students, were primarily responsible for one student obtaining less Post-test marks.

The researcher has highlighted the research done in mathematics when they help improve student's ability to problem solving questions using different models, common difficulties that students face in problem solving questions, the definition of problem solving itself in mathematics and Polya's problem solving model and also describes the methods used to determine if the students have increased in their problem solving level by the usage of Polya's problem-solving model.

The findings from the student's performances in the Pre-test and Post-test scores bring out the following:

- Improvement in scores by following Polya's model.
- Better scores relate to better understanding of the word problems.
- Better scores also are indicative of students' ability to understand the inter-related aspects in the problem.

Acknowledgements

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